Paper Reference(s)

6678

Edexcel GCE

Mechanics M2

Advanced Level

Friday 25 January 2013 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. Two uniform rods AB and BC are rigidly joined at B so that $\angle ABC = 90°$. Rod AB has length 0.5 m and mass 2 kg. Rod BC has length 2 m and mass 3 kg. The centre of mass of the framework of the two rods is at G.

(*a*) Find the distance of G from BC.

(2)

The distance of G from AB is 0.6 m.
The framework is suspended from A and hangs freely in equilibrium.

(*b*) Find the angle between AB and the downward vertical at A.

(3)

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2. A lorry of mass 1800 kg travels along a straight horizontal road. The lorry's engine is working at a constant rate of 30 kW. When the lorry's speed is 20 m s^{-1}, its acceleration is 0.4 m s^{-2}. The magnitude of the resistance to the motion of the lorry is R newtons.

(*a*) Find the value of R.

(4)

The lorry now travels up a straight road which is inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{12}$. The magnitude of the non-gravitational resistance to motion is R newtons. The lorry travels at a constant speed of 20 m s^{-1}.

(*b*) Find the new rate of working of the lorry's engine.

(5)

3.

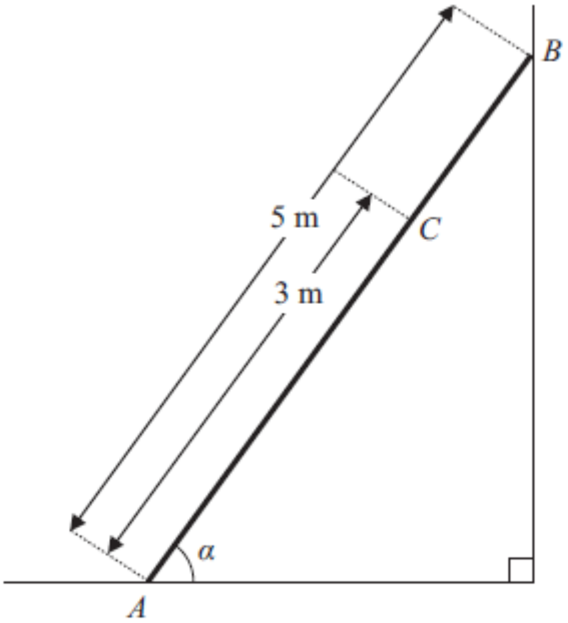

Figure 1

A ladder, of length 5 m and mass 18 kg, has one end *A* resting on rough horizontal ground and its other end *B* resting against a smooth vertical wall. The ladder lies in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{4}{3}$, as shown in Figure 1. The coefficient of friction between the ladder and the ground is μ. A woman of mass 60 kg stands on the ladder at the point *C*, where $AC = 3$ m. The ladder is on the point of slipping. The ladder is modelled as a uniform rod and the woman as a particle.

Find the value of μ.

(9)

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4. At time *t* seconds the velocity of a particle *P* is $[(4t - 5)\mathbf{i} + 3\mathbf{j}]$ m s^{-1}. When $t = 0$, the position vector of *P* is $(2\mathbf{i} + 5\mathbf{j})$ m, relative to a fixed origin *O*.

(*a*) Find the value of *t* when the velocity of *P* is parallel to the vector **j**.

(1)

(*b*) Find an expression for the position vector of *P* at time *t* seconds.

(4)

A second particle *Q* moves with constant velocity $(-2\mathbf{i} + c\mathbf{j})$ m s^{-1}. When $t = 0$, the position vector of *Q* is $(11\mathbf{i} + 2\mathbf{j})$ m. The particles *P* and *Q* collide at the point with position vector $(d\mathbf{i} + 14\mathbf{j})$ m.

(*c*) Find

(i) the value of *c*,

(ii) the value of *d*.

(5)

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5. The point A lies on a rough plane inclined at an angle θ to the horizontal, where $\sin \theta = \frac{24}{25}$. A particle P is projected from A, up a line of greatest slope of the plane, with speed U m s^{-1}. The mass of P is 2 kg and the coefficient of friction between P and the plane is $\frac{5}{12}$. The particle comes to instantaneous rest at the point B on the plane, where AB =1.5 m. It then moves back down the plane to A.

(*a*) Find the work done against friction as P moves from A to B.

(4)

(*b*) Use the work-energy principle to find the value of U.

(4)

(*c*) Find the speed of P when it returns to A.

(3)

6.

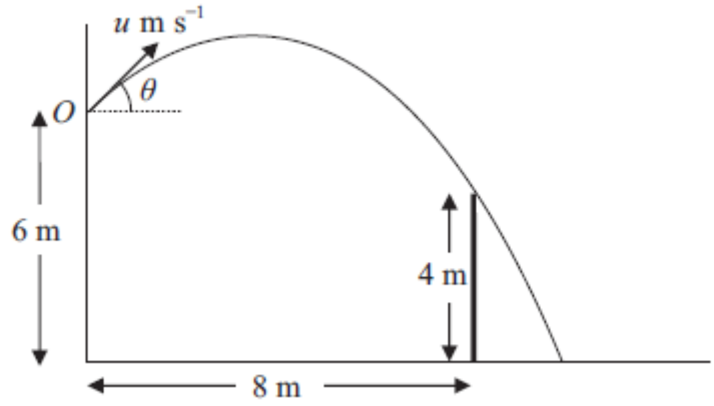


Figure 2

A ball is thrown from a point O, which is 6 m above horizontal ground. The ball is projected with speed u m s^{-1} at an angle θ above the horizontal. There is a thin vertical post which is 4 m high and 8 m horizontally away from the vertical through O, as shown in Figure 2. The ball passes just above the top of the post 2 s after projection. The ball is modelled as a particle.

(*a*) Show that $\tan \theta = 2.2$.

(5)

(*b*) Find the value of u.

(2)

The ball hits the ground T seconds after projection.

(*c*) Find the value of T.

(3)

Immediately before the ball hits the ground the direction of motion of the ball makes an angle α with the horizontal.

(*d*) Find α.

(5)

7. A particle A of mass m is moving with speed u on a smooth horizontal floor when it collides directly with another particle B, of mass $3m$, which is at rest on the floor. The coefficient of restitution between the particles is e. The direction of motion of A is reversed by the collision.

(*a*) Find, in terms of e and u,

(i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

(7)

After being struck by A the particle B collides directly with another particle C, of mass $4m$, which is at rest on the floor. The coefficient of restitution between B and C is $2e$. Given that the direction of motion of B is reversed by this collision,

(*b*) find the range of possible values of e,

(6)

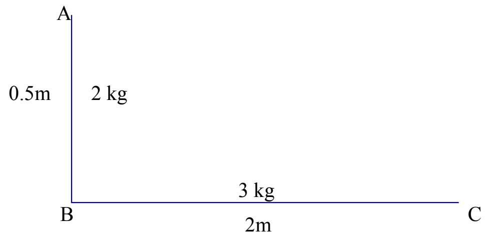
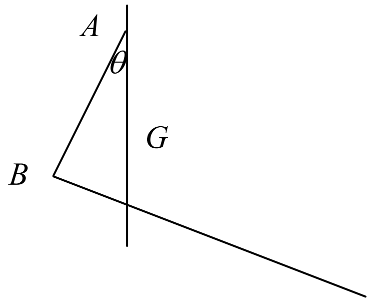
(*c*) determine whether there will be a second collision between A and B.

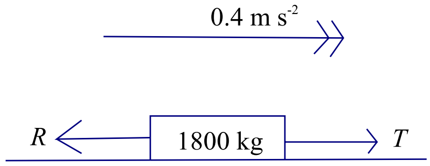
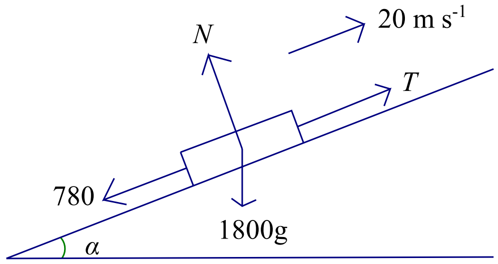
(3)

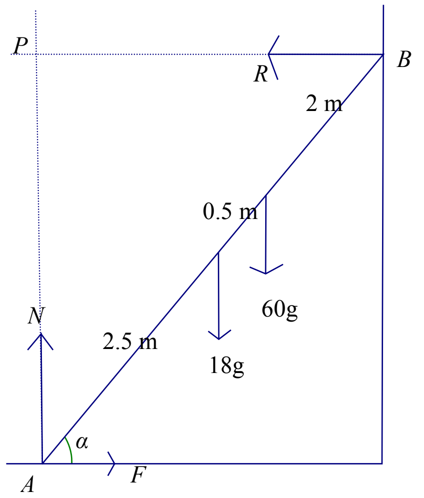
TOTAL FOR PAPER: 75 MARKS

END

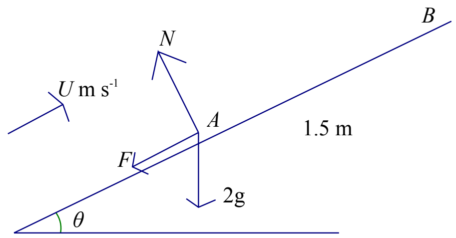
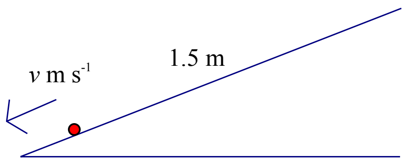
January 2013
6678 M2
Mark Scheme

Q.	Scheme	Marks	
1. (a)		M1	
	$5\bar{y} - 2\times 0.25(+0)$	A1	Moments equation with lengths ¼, 1 and (ratio of) masses 2, 3. Allow moments about a parallel axis Use of length for mass is M0.
	$\bar{y} = \dfrac{2\times 0.25}{5} = 0.1$		For distance from *BC*
(b)			
		M1	
	$\tan\theta = \dfrac{0.6}{0.5-0.1}$	A1ft	Must suspend from *A*. Use of tan with 0.6 and $0.5-\bar{y}$ Could be wrong way up. Must be using 0.6
		A1	Correct way up. ft their $\bar{y}$.
	$\theta = \tan^{-1}\left(\dfrac{6}{4}\right) = 56.3° = 56°$		Accept awrt 56.3

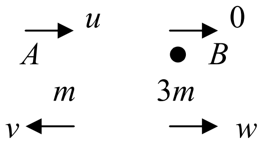
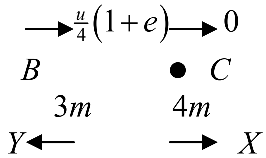
Q.	Scheme	Marks	
2 (a)		B1	
	$T = \frac{30000}{20} \quad (=1500)$	M1	Use of $P = Fv$
	$T - R = 1800a$	A1	Equation of motion. Need all 3 terms. Condone sign errors
	$T - R = 1800 \times 0.4$		Equation correct (their T)
	$R = 1500 - 1800 \times 0.4$	A1	
	$= 780$		Only
(b)		M1	
	$T - 1800g\sin\alpha - R = 0$	A1	Equation of motion. Need all 3 terms. Weight must be resolved. Condone cos for sin. Condone sign errors Correct equation. Allow with *R* not substituted or with their *R*.
	$T = 1800 \times \frac{1}{12}g + 780$	DM1	
	Power $= \left(1800 \times \frac{1}{12}g + 780\right) \times 20$	A1	Use of $P = Tv$
		A1	Correctly substituted equation (for their *R*)
	$= 45000$ W or 45 kW		cao

Q	Scheme	Marks	
3			
	$F = \mu N$	B1	Used. Condone an inequality.
	R(↑) $18g + 60g = N$	M1	Resolve vertically
	$=78g$	A1	
	R(→) $R = F = \mu N$		
P	$2.5 \times 18g\cos\alpha + 3 \times 60g\cos\alpha = 5F\sin\alpha$	M1A2	Moments equation. Condone sign errors. Condone sin/cos confusion -1 each error
A	$18g \times 2.5\cos\alpha + 60g \times 3\cos\alpha = R \times 5\sin\alpha$		
C	$\frac{1}{2}\cos\alpha \times 18g + 3\sin\alpha F + 2\sin\alpha R = 3\cos\alpha N$		
B	$5\cos\alpha N = 5\sin\alpha F + 2.5\cos\alpha \times 18g + 2\cos\alpha \times 60$		
W	$60g \times \frac{1}{2}\cos\alpha + 2.5N\cos\alpha = 2.5R\sin\alpha + 2.5F\sin\alpha$		
	$45 \times \frac{3}{5}g + 180 \times \frac{3}{5}g = 4R$	DM1	Eliminate α. Dependent on the second M1.
	$R = \frac{135}{4}g$		
	$78g\mu = \frac{135}{4}g$	DM1	Equation in μ only. (Dependent on the first two M marks.)
	$\mu = \frac{135}{4 \times 78} = \frac{135}{312} = 0.432... = 0.43$	A1	NB g cancels. 0.43269..., $\frac{225}{520}$, $\frac{45}{104}$, awrt 0.433 Do not accept an inequality.
	NB If use just two moments equations, M1A2 for the better attempt, M1A1 for the other. Remaining marks as above.		
4		B1	

Q	Scheme	Marks	
(a)	$t = \frac{5}{4}$	M1	1.25
(b)	$\mathbf{r} = \left(2t^2 - 5t\right)\mathbf{i} + 3t\mathbf{j}\left(+\mathbf{c}\right)$		Integrate the velocity vector
		A1	NB Also correct to use suvat with $\boldsymbol{a}$ = 4**i** and $\boldsymbol{u}$ = -5**i** + 3**j**.
		DM1	Correct
	$t = 0 \quad 2\mathbf{i} + 5\mathbf{j} = \mathbf{c}$	A1	Use $\mathbf{r}_0$ to find C
	$\mathbf{r} = \left(2t^2 - 5t\right)\mathbf{i} + 3t\mathbf{j} + \left(2\mathbf{i} + 5\mathbf{j}\right)$		oe
	$(2t^2 - 5t + 2)\mathbf{i} + (3t + 5)\mathbf{j}$	B1	
(c)	$\mathbf{r}_Q = 11\mathbf{i} + 2\mathbf{j} - 2t\mathbf{i} + ct\mathbf{j}$		Correct **j** component of $\mathbf{r}_Q$
	$(11 - 2t)\mathbf{i} + (2 + ct)\mathbf{j}$		Do not actually require the whole thing - can answer the Q by considering only the **j** component.
	$\mathbf{r}_P = \left(2t^2 - 5t + 2\right)\mathbf{i} + \left(3t + 5\right)\mathbf{j}$		
	$\mathbf{r}_Q = \mathbf{r}_P = d\mathbf{i} + 14\mathbf{j}$	$2t^2 - 5t$	
	$3t + 5 = 14$ $t = 3$ (alternative: $2t^2 - 3t - 9$ $(2t + 3)(t - 3) = 0$ $t = 3$ A1 ft)	M1 A1	Form an equation in t only
	$2 + ct = 14 \Rightarrow c = 4$	A1 ft	Their t
	$d = 11 - 2 \times 3 = 5$ or $d = 2 \times 3^2 - 5 \times 3 + 2 \Rightarrow d = 5$		Their t
	Alt: $2t^2 - 5t + 2 = 11 - 2t = d \Rightarrow t = \frac{11 - d}{2}$ $2\left(\frac{11-d}{2}\right)^2 - 5\left(\frac{11-d}{2}\right) + 2 = d$, $d^2 - 19d + 70 = 0 = (d - 5)(d - 14)$		

Q.	Scheme	Marks	
5	 		
(a)	$N = 2g\cos\theta = \frac{14}{25}g$	M1	Resolve perpendicular to plane. Condone trig confusion.
	$F = \mu N = \frac{5}{12}\times\frac{14}{25}g = \frac{7g}{30}$	B1	Correct value of F seen or implied
	Work done $= \frac{7}{30}g\times 1.5 = 3.43... = 3.4$ J	DM1	Their F x 1.5
		A1	$\frac{7g}{20}$, 3.4 or 3.43 only
(b)	$3.43 + 2g\sin\theta\times 1.5 = \frac{1}{2}\times 2U^2$	M1	Energy equation - needs all 3 terms, but condone sign errors & trig confusion. Must have an expression for the vertical height.
		A1	Correct with one slip for their WD.
		A1	All correct for their WD
	$U = 5.626.... = 5.6$	A1	5.6 & 5.63 only
(c)	 		
	$2g\sin\theta\times 1.5 = 3.43 + \frac{1}{2}\times 2v^2$	M1	Energy equation - needs all three terms. Condone sign errors & trig. confusion. Extra terms give M0.
	OR: $\frac{1}{2}\times 2U^2 = 2\times 3.43 + \frac{1}{2}\times 2v^2$	A1	All correct (their WD & U)
	$v^2 = 3g\sin\theta - 3.43$ $v = 4.979...$ Speed = 5.0 m s^{-1}	A1	Accept 4.98
Alt (c)	$mg\sin\theta - F = ma$ and $v^2 = (u^2) + 2as$	M1	Equation of motion - needs all three terms. Condone sign errors & trig. confusion. Together with *suvat*
	$2g\sin\theta - \frac{7g}{30} = \frac{48g}{25} - \frac{7g}{30} = 2a$ $a = \frac{253g}{300} = 8.26....$	A1	
	$v^2 = 24.794$, $v = 5.0$	A1	Accept 4.98

Q.	Scheme	Marks	
6 (a)	$2 = -2u\sin\theta + \frac{1}{2}g \times 4$	M1	Vertical distance. Condone sign errors. Must have used $t = 2$, but could be using $u_y = u\sin\theta$
	$\left(-2 = u\sin\theta t - \frac{1}{2}gt^2\right)$	A1	All correct
	$u\sin\theta = g - 1$		
	$2u\cos\theta = 8 \quad (u\cos\theta = 4)$ $(u\cos\theta t = 8)$	B1	Horizontal distance. Accept $u_x = 4$ o.e.
	$\tan\theta = \frac{g-1}{4} = 2.2$ *	M1	Divide to obtain expression for $\tan\theta$
		A1	**Given answer** It is acceptable to quote and use the equation for the projectile path. Incorrect equation is 0/5.
(b)	$u\cos\theta = 4$	M1	Use the horizontal distance and θ to find u
	$u = \frac{4}{\cos\theta} = 9.66... = 9.7$	A1	9.67 or 9.7 NB $\theta = 65.6°$ leading to 9.68 is an accuracy penalty.
	OR use components from (a) and Pythagoras.		
(c)	$6 = (1-g)T + \frac{1}{2} \times 9.8T^2$	M1	Equation for vertical distance $= \pm 6$ to give a quadratic in T. Allow their u_y
	$4.9T^2 - 8.8T - 6 = 0$		
	$T = \frac{8.8 \pm \sqrt{[(-)8.8]^2 + 24 \times 4.9}}{9.8}$	DM1	Solve a 3 term quadratic
	$T = 2.323... = 2.32$ or 2.3	A1	2.3 or 2.32 only
(d)	$v^2 = 8.8^2 + 2g \times 6$ or $v = -8.8 + gT$	M1	Use *suvat* to find vertical speed
		A1	Correct equation their u_y, T
	$v = 13.96...$		
	Horiz speed = 4		
	$\tan\alpha = \frac{v}{4}$	DM1	Correct trig. with their vertical speed to find the required angle.
		A1	Correct equation
	$\alpha = 74.01... = 74°$	A1	$74°$ or $74.0°$. Allow 106.
	Alternative:		
	$\frac{1}{2}m(9.6664)^2 + 6mg = \frac{1}{2}mv^2$	M1	Conservation of energy to find speed
	$v = 14.52719...$	A1	
	$\cos\alpha = \frac{4}{14.5}$	DM1	Correct method for α
		A1	
	$\alpha = 74.01... = 74°$	A1	Allow 106

Q	Scheme	Marks	
7(a)			If the signs on their diagram and in their working are inconsistent, ignore the diagram. Penalise inconsistency between the two equations in the second accuracy mark.
	$mu = -mv + 3mw$	M1	CLM. Allow for v in either direction. Needs all 3 terms. Condone sign errors.
	$u = -v + 3w$	A1	v in either direction. Ignore diagram if equations "correct" but inconsistent with diagram.
	$eu = w + v$	M1	Impact law. Must be the right way round, but condone sign errors
		A1	Correct equation. Signs consistent with CLM equn.
	$w = \frac{u}{4}(1+e)$	DM1	Solve for v or w.
		A1	One correct
	$v = -w + eu = \frac{u}{4}(3e-1)$	A1	Both correct. $1-3e \rightarrow$ A0 for v
(b)			If the signs on their diagram and in their working are inconsistent, ignore the diagram. Penalise inconsistency between the two equations in the B mark.
	$3mw = 4mX - 3mY$	M1	CLM for their w.
		A1ft	Correct unsimplified (their w)
	$2ew = X + Y$	B1ft	Impact law. Must be the right way up. Their w
	$7Y = W(8e-3)$	DM1	Solve for $(7)Y$
	Or $2ue(1+e) - \frac{3u}{4}(1+e) = 7Y$		
	$\rightarrow e > \frac{3}{8}$	A1	
	$Y > 0 \rightarrow \frac{3}{8} < e \leq \frac{1}{2}$	A1	NB No longer ft. Condone <.
(c)	$\frac{u}{28}(1+e)(8e-3) > \frac{u}{4}(3e-1)$	M1	For a second collision their $Y >$ their v
	$2e^2 - 4e + 1 > 0$		
	$e = \frac{4 \pm \sqrt{16-8}}{4} = 1.707,\ 0.293$	DM1	Obtain the critical values
	$2e^2 - 4e + 1 < 0$ for $\frac{3}{8} < e \leq \frac{1}{2}$ so no second collision.	A1	Compare 0.293 (o.e.) with $\frac{3}{8}$ to reach correct conclusion for correct reason.